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COSMOLOGY AND GALACTIC ROTATION CURVES

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We explore the possibility that the entire departure of galactic rotational velocities from their luminous Newtonian expectation be cosmological in origin, and show that within the framework of conformal gravity (but not Einstein gravity apparently) every static observer sees the overall Hubble flow as a local universal linear potential which is able to account for available data without any need for dark matter. We find that the Universe is necessarily an open one with 3-space scalar curvature given by $k = -3.5 \times 10^{-60}\text{cm}^{-2}$.

At the present time the search for galactic dark matter stands at an extremely critical juncture, with neither the recent gravitational microlensing observations of the OGLE, MACHO and EROS collaborations or the optical searches of the recently refurbished Hubble Space Telescope having been able to confirm the existence of the copious amounts of dark or faint matter that had been widely surmised to reside in the spherical haloes of galaxies such as the Milky Way. At the very minimum one can say that these searches have certainly not yet achieved their intended goal of confirming the standard Newton-Einstein dark matter picture, while at the maximum one can say that they have even thrown the entire picture into question.

Now while the standard theory is the clear preference of the bulk of the astrophysical community, nonetheless, a small set of authors have ventured (long before the microlensing searches in fact) to suggest that the dark matter problem lies not in our ignorance of the matter content of galaxies but rather in our reliance on the use of Newton's Law of Gravity on distance scales much larger than the solar system ones on which it was first established; with Milgrom, Bekenstein and Sanders (who all have explored the MOND alternative) and Mannheim and Kazanas (with their conformal gravity) having been perhaps the most persistent critics of the standard paradigm. What mainly distinguishes the conformal gravity program (viz. gravity based on the conformal invariant fourth order gravitational action $I_W = -\alpha \int d^4x(-g)^{1/2} C_{\lambda\mu\nu\kappa} C^{\lambda\mu\nu\kappa}$

where $C_{\lambda\mu\nu\kappa}$ is the conformal Weyl tensor) from other alternative approaches is that it sets out to generalize not the Newtonian potential, but rather the Schwarzschild solution, so that from the outset the theory is fully covariant and fully relativistic. Indeed, exterior to a static, spherically symmetric source, the exact vacuum metric of the theory takes the form (Mannheim and Kazanas 1989) $ds^2 = B(r)c^2dt^2 - B(r)dr^2 - r^2d\Omega$ where $B(r) = 1 - 2\beta/r + \gamma r$, to thus nicely recover both the Schwarzschild solution and its associated Newtonian potential, thereby enabling the theory to still meet the classic General Relativity tests; with the new linear potential term then only providing corrections to Newton on large rather than on small scales.

For galaxies, once we make the standard Newtonian assumption that the galaxies can be treated as isolated objects (an issue we will in fact have to reconsider below), the application of the above metric to rotation curves is straightforward. We simply integrate the individual stellar potentials $V^*(r) = -\beta^*c^2/r + \gamma^*c^2r/2$ over all the N^* stars (and interstellar gas) in each galaxy, taking the stars to be distributed just as the detected light and normalized to it with a (galaxy dependent) mass to light ratio M/L. Now while the ensuing fits (see Mannheim 1993 and the more detailed unpublished fitting of Mannheim and Kmetko and of Carlson and Lowenstein) are acceptable as far as the shape of the rotation curves is concerned (fits in fact almost completely identical to those of Fig. (1) which we will present below), the fits were only able to match the normalizations of the curves provided the quantity $N^*\gamma^*$ was close to a universal, galaxy independent value of order 10^{-30}cm^{-1}. Thus rather than γ^* being found to be universal, it was the total galactic linear potential coefficient $\gamma_{gal} = N^*\gamma^*$ which came out universal instead. Thus unless some reason can be found for why the effective γ^* adjusts itself each and every time to the total luminosity in each galaxy rather than the other way round, this possible explanation of galactic rotation curves would have to be set aside.

While the very application of $V^*(r)$ to the data thus shows that by itself it does not in fact work, nonetheless we find that it fails in a very instructive way, namely it reveals that the linear potential fits do capture the essence of the data, and that they have to be normalized in a manner which is essentially independent of the matter content of each individual galaxy with a magnitude which turns out numerically to be close to that of the inverse Hubble radius, a number of cosmological significance. The structure found for the fits thus suggests that there might indeed be some universal linear potential, but that rather than its being due to summing over all the stars in a given galaxy, it would instead have to be due to the effect of all

of the other galaxies in the Universe on a given one. To numerically motivate this possibility we calculate the magnitude of the centripetal acceleration v^2/c^2R at the data point farthest from the center of each of the 11 sample galaxies listed in Table (1). (This particular set of galaxies was identified by Begeman, Broeils and Sanders 1991 (their paper gives complete data references) as being a particularly reliable set of HI rotation curve data, and we use the same galactic input parameters (distance, luminosity, stellar disk scale length, mass of HI gas) in Table (1) as they did, except for NGC2841 which uses the adopted distance subsequently suggested by Sanders and Begeman 1994.) As we can see, despite a variation of a factor of 1000 or so in luminosity, the farthest v^2/c^2R only vary by a factor of 2 or so around a mean value of $3.1 \times 10^{-30}\mathrm{cm}^{-1}$. (Because the farthest points in DDO154 may be affected by random gas pressures, for it we simply take the value at the point with the highest velocity). The data thus suggest a universal centripetal acceleration, and thus a universal linear potential. Indeed, independent of our own interest here, this would appear to be an interesting regularity in and of itself.

Now in the first approximation the galaxies in the Universe are distributed smoothly in a homogeneous and isotropic Robertson-Walker (RW) geometry which would not initially appear to have much connection to a linear potential, and indeed in Einstein gravity there does not appear to be one as far as we can tell. Now the RW geometry is also a solution to the cosmology associated with conformal gravity (Mannheim 1992). However, since the Weyl tensor vanishes in a RW geometry (the geometry being conformal to flat), in the conformal theory not only is RW a solution but so also is the RW metric multiplied by any overall conformal factor (a factor which is unobservable because of the underlying conformal invariance itself). Now in their original paper, Mannheim and Kazanas (1989) noted the kinematic fact that under the general coordinate transformation

$$\rho = 4r/(2(1+\gamma_0 r)^{1/2} + 2 + \gamma_0 r) \qquad , \qquad \tau = \int R(t)dt \tag{1}$$

we can effect the metric transformation

$$ds^2 = (1+\gamma_0 r)c^2dt^2 - dr^2/(1+\gamma_0 r) - r^2 d\Omega$$

$$\rightarrow \frac{1}{R^2(\tau)} \frac{(1+\rho\gamma_0/4)^2}{(1-\rho\gamma_0/4)^2} \left(c^2 d\tau^2 - \frac{R^2(\tau)}{(1-\rho^2\gamma_0^2/16)^2}(d\rho^2 + \rho^2 d\Omega) \right) \tag{2}$$

to yield a metric which is conformal to a RW metric with scale factor $R(\tau)$ and (explicitly negative) 3-space scalar curvature $k = -(\gamma_0/2)^2$. (In passing we note that in the cosmology discussed in Mannheim 1992 an open Universe with very negative k was in fact realized, with such a Universe not suffering from the flatness problem found in the standard cosmology). Now, and this is the key point, in a geometry which is both homogeneous and isotropic about all points, all observers can use the same position independent conformal time τ, and any observer can serve as the origin for the coordinate ρ; thus in his own local rest frame each observer is able to make the general coordinate transformation of Eq. (1) involving his own particular ρ. Moreover, since the observer is also free in the conformal theory to make arbitrary conformal transformations as well, that observer will then be able to see the entire Hubble flow appear in his own local static coordinate system as a universal linear potential with a universal acceleration $\gamma_0 c^2/2$ coming from the spatial curvature of the Universe. Now in that specific static coordinate system any other Hubble flow observer would see something entirely different and not recognize anything that would look like a simple universal linear potential at all. Only in his own explicit rest frame would any other observer be able to recognize such a universal linear potential. However, while the transformations of Eqs. (1) and (2) would not be useful for describing the Hubble flow motions of the individual galaxies themselves, they appear to be ideally suited for describing the internal orbital motions of the stars and gas within each galaxy, since each internal motion can be discussed independently in each galaxy's own rest frame. Thus it would appear that in conformal gravity each observer sees the general Hubble flow metric as a local universal linear potential with a strength fixed by the scalar curvature of the Universe (a time independent quantity unlike the time dependent Hubble parameter itself), with the matter in each galaxy now acting as test particles which are being swept through the Hubble flow. (In passing we note the explicit role played here by curved space. In strictly Newtonian physics the only effect of any background would be to put tidal forces on individual galaxies, forces that would not account for the rotational motions of stars and gas but only to a departure therefrom. What we find here is that the Hubble flow accounts for the explicit motions of test particles around the center of the galaxies rather than to a tidal perturbation to that motion. Given also the fact that linear potentials are not asymptotically flat, we thus see that in curved space Newtonian reasoning can be completely misleading.)

In order to now apply this cosmological effect to explicit galactic motions, we must look not just at the background RW metric but rather at the embedding of each galaxy treated as a local inhomogeneity in that background, inhomogeneities which in the absence of the background are already putting out potentials such as those generated by V^* above. Now we have not been able to find an exact solution to this embedding problem, so for weak gravity we shall simply add the universal cosmological potential of Eq. (2) onto that generated by the V^* potential. However, if the cosmological background is to be responsible for the regularity found for the galactic v^2/c^2R, we should then expect the cosmological γ_0 to be of order 10^{-30}cm^{-1}. Consequently, since 10^{12} or so galaxies make up the entire visible Universe, each individual galactic $\gamma_{gal} = \gamma^* N^*$ would then be of order 10^{-42}cm^{-1} or so and thus completely irrelevant locally. Hence the only local galactic term which is relevant is the standard Newtonian one with resulting acceleration g_N, with the entire local motion then being described by

$$v^2/R = g = g_N + c^2\gamma_0/2 \tag{3}$$

in first approximation. Now the Universe may not be exactly RW since it appears to possess inhomogeneities on the largest scales (suggesting that the scalar curvature may have some variation on large scales). Also in making the transformation of Eq. (1) we ignore any local inhomogeneities (and in particular any non-spherically symmetric ones) as well, so while we might initially expect γ_0 to be completely universal, we note that for fitting purposes we may anticipate some small variation in the fits.

We thus now apply Eq. (3) to our 11 galaxies, and initially fit each galaxy with its own γ_0 and its own mass to light ratio M/L to find the fits of Fig. (1). (The full line gives the overall fit to each galaxy, the dashed line the pure Newtonian contribution, and the dash dot line the pure linear contribution). As we can see from the fits, the linear potential model fits the data quite well with the pure linear contribution completely mirroring the dark matter halo contribution familiar from the standard model, save only that the linear potential must eventually cause the rotation curves to rise even while it makes then flat in the observed region. (This key feature could eventually enable one to distinguish between the linear potential theory and theories such as isothermal haloes or MOND (Milgrom 1983) which require the flatness to be an asymptotic rather than merely an intermediate property of the rotation curves). The derived values for

γ_0 and M/L are listed in Table (1), and as we see, the derived values for the γ_0 are indeed very close, to within a factor of 2 about a mean value. The data are thus not rejecting Eq. (3) out of hand. To explore the variation found for γ_0, we also made a fit to the complete set of the 11 galaxies using just one selfsame overall $\gamma_0 = 3.75 \times 10^{-30}\text{cm}^{-1}$ for the whole sample to yield the dotted curves in Fig. (1). While not giving spot on fitting, we see that this fitting gets to within 10% or so of each data point, viz. much less than the factor of 2. Finally, if we allow the adopted distances to each galaxy to vary by up to 25% or so we could then bring the dotted curves down to the full curve fits. (As such our fits are on a par with those of MOND which possesses its own universal acceleration a_0, with Begeman, Broeils and Sanders 1991 finding a precisely similar pattern for MOND fits to the same galaxies, viz. universal a_0 to within a factor of 2 galaxy by galaxy, or strictly universal a_0 with instead an up to 20% or so variation in the adopted distances.) Our fitting should thus be regarded as acceptable and competitive with both MOND and the standard dark matter models. Of course, beyond the phenomenological issue, unlike either MOND or dark matter, our Eq. (3) is a fully motivated output to a fully covariant theory rather than being merely a phenomenologically motivated input, and for that reason alone Eq. (3) is already to be preferred over the other contenders. Moreover, if our theory is in fact correct, then it provides us with an actual measurement of the scalar curvature of the Universe, something which despite years of intensive work has yet to be achieved in the standard theory.

It is of some interest to identify why it is that the linear potential theory gives flat rotation curves at all in the observed region rather than ones that rise right away. The answer to this lies in a regularity first noted by Freeman, namely that the most prominent spiral galaxies all seem to have a common central surface brightness, Σ_0^F. (In passing we note that while there also exist low surface brightness galaxies with $\Sigma_0 < \Sigma_0^F$, there do not appear to be any galaxies with $\Sigma_0 > \Sigma_0^F$, thus making Σ_0^F a so far unexplained upper bound on galaxies). Additionally, the Freeman limit galaxies (galaxies which include all the bright galaxies in our 11 galaxy sample) all seem to obey the universal Tully-Fisher law, a phenomenologically established universal relation between the luminosity and the fourth power of the velocity dispersion in the observed flat rotation curve region. Bright galaxies thus possess a great deal of universality. For an exponential disk spiral ($\Sigma(R) = \Sigma_0 \exp(-R/R_0)$) we thus now note that since the pure Newtonian contribution causes the rotation curve to peak at around $2R_0$ with a normalization which depends on Σ_0, we can then universally match

γ_0 to Σ_0^F for the Freeman limit galaxies so that the value of the velocity at around $10R_0$ or so (a region where the linear term dominates) will be equal to its value at the $2R_0$ Newtonian peak in the Freeman limit galaxies. Further, at around $5.5R_0$ the Newtonian contribution has dropped to about half its peak value, while the linear contribution is about half of its value at $10R_0$, to thus give the total velocity at $5.5R_0$ a magnitude equal to its values at both $2R_0$ and $10R_0$, and thus a flat rotation curve from $2R_0$ all the way out to about $10R_0$ after which the ultimate rise required of the linear potential must begin to set in. Unlike dark matter fits where the halo parameters have to be varied galaxy by galaxy, in conformal gravity flatness is thus universally achieved with no need for any adjustment of parameters. Further, since we have tuned γ_0 to Σ_0^F, at around $10R_0$ or so, the velocity there obeys $v^4 \sim R_0^2(\gamma_0)^2 \sim R_0^2(\Sigma_0^F)^2 \sim \Sigma_0^F L$, which we recognize as the Tully-Fisher relation. The universal matching of Σ_0^F and γ_0 thus leads to both flatness and Tully-Fisher. (Since we have now matched γ_0 to Σ_0^F, for the gas rich, low surface brightness galaxies where $\Sigma_0 < \Sigma_0^F$, it follows that their rotation curves should simply start rising right away, a trend which is in fact apparent in the data. In fact this trend is the analog of the trend found in dark matter fits where the lower luminosity galaxies are found to be proportionately darker.) It is important to note that we have not in fact provided an ab initio explanation for the Tully-Fisher relation since we have not yet explained why there is in fact a Freeman limit in the first place. However, since we have now correlated Σ_0^F with the cosmologically based γ_0, this suggests that the Freeman limit may arise as upper bound on the galaxies which are generatable as fluctuations out of the cosmological background, a background which is indeed controlled by the Hubble scale. The establishing of such a cosmological origin for Σ_0^F would then provide a complete a priori derivation of the Tully-Fisher relation and of the systematics of rotation curves which we have presented here.

Our uncovering of an apparent universal acceleration in conformal gravity immediately recalls the presence of a similar feature in Milgrom's MOND, despite the fact that our motivation is entirely different. Specifically, Milgrom had suggested that if a universal acceleration a_0 did exist, then Newton's Second Law could possibly be modified into a relation with a form such as $\mu(g/a_0)g = g_N$. The candidate functional form $\mu(x) = x/(1+x^2)^{1/2}$ then yields

$$g = g_N\{1/2 + (g_N^2 + 4a_0^2)^{1/2}/2g_N\}^{1/2} \tag{4}$$

an expression which is found to perform extremely well phenomenological despite the absence of any deeper underlying theory. Unlike Eq. (3), Eq. (4) will lead to asymptotically flat rotation curves, and is thus quite distinct from Eq. (3), though it is of interest to note that Eq. (3) would in fact follow from the general MOND approach if the function $\mu(x)$ were instead to take the form $\mu(x) = 1 - 1/x$. Conformal gravity thus not only provides a rationale for why there is in fact a universal acceleration in the first place (something simply assumed in MOND) but also yields an explicit form for the function $\mu(x)$, albeit not the one previously considered in MOND studies. Also of course, in conformal gravity the universal acceleration is obtained from an equally universal (linear) potential, something which is not the case in MOND. Now we have noted that both Eqs. (3) and (4) both seem to fit the available data equally well, and it would be quite remarkable if two different formulas both worked. However, it turns out that there is a reason why they both work, namely the existence of the Freeman limit to which we referred above, a limit which forces Freeman limit galaxies with a common mass to light ratio to automatically obey the common mass-radius relation $M = 2\pi(M/L)\Sigma_0^F R_0^2$. Now since asymptotically the MOND formula of Eq. (4) yields $g = (MGa_0)^{1/2}/R$, the value of this quantity would then numerically actually agree with that of Eq. (3) (viz. $g = c^2\gamma_0/2$) at around $R = 10R_0$ (i.e. at the end of the flat region in the fits of Fig. (1)) simply because of the validity of this mass-radius relation. Since the MOND formula is also flat all the way down to $2R_0$ for Freeman limit galaxies due to the way the MOND function $\mu(x) = x/(1+x^2)^{1/2}$ interpolates back from the $10R_0$ region, we see that Eqs. (3) and (4) must agree for Freeman limit galaxies over the entire $2R_0$ to $10R_0$ region, even as they radically disagree at larger distances. (Though we did not detect any differences in our particular fits, detailed analysis of a larger sample of sub-Freeman limit galaxies might eventually provide some discrimination between Eqs. (3) and (4).)

As regards these mass-radius and Tully-Fisher relations, we note that they apply not only to bright galaxies but also (in an appropriate form) to globular clusters and to clusters of galaxies as well, a feature which had been noted by Kazanas and Mannheim (1991) and examined in detail by Schaeffer et al (1993). In particular Fig. (4) of Schaeffer et al shows that the quantity v^2R/L is essentially universal over an enormous luminosity range from 10^4 to 10^{12} solar luminosities. Thus $v^2/R \sim L/R^2$ over the same range, i.e. the entire range is driven by effectively a single mean surface brightness Σ_0^F and a single universal acceleration, to thus

enable conformal gravity to immediately also give acceptable values for the velocity dispersions in clusters of galaxies again without needing any dark matter. Finally, we recall that Kazanas and Mannheim (1991) also noted that the mass-radius relation even appears to apply to the entire Universe itself (essentially because the Schwarzschild radius of the entire Universe is of order the Hubble radius), and, curiously even to a single elementary particle as well. Thus we believe it is possible to make a case for the existence of a universal linear potential associated with the cosmological Hubble flow, an intriguing possibility which appears to eliminate the need for dark matter.

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Figure Caption

Figure (1). The calculated rotational velocity curves associated with the conformal gravity potential of Eq. (3) for each of the 11 galaxies in the sample. In each graph the bars show the data points with their quoted errors, the full curve shows the overall theoretical velocity prediction (in km sec^{-1}) as a function of distance from the center of each galaxy (in units of R/R_0 where each time R_0 is each galaxy's own scale length) obtained by allowing each galaxy's acceleration parameter to vary independently, while the dashed and dash-dotted curves show the velocities that the Newtonian and linear potentials would then separately produce. The dotted curve shows the velocities that would be produced by a completely galaxy independent mean universal acceleration.

Table (1)

Galaxy	*Distance*	*Luminosity*	R_0	M_{HI}	v^2/c^2R	$(M/L)_{disk}$	$\gamma_0/2$
	(Mpc)	$(10^9 L_{B\odot})$	(kpc)	$(10^9 M_\odot)$	$(10^{-30}\mathrm{cm}^{-1})$	$(M_\odot/L_{B\odot})$	$(10^{-30}\mathrm{cm}^{-1})$
DDO 154	4.00	0.05	0.50	0.27	1.44	0.60	1.26
DDO 170	12.01	0.16	1.28	0.45	1.63	4.92	1.44
NGC 1560	3.00	0.35	1.30	0.82	2.70	1.62	2.34
NGC 3109	1.70	0.81	1.55	0.49	1.98	0.03	1.63
UGC 2259	9.80	1.02	1.33	0.43	3.85	3.80	2.27
NGC 6503	5.94	4.80	1.73	1.57	2.14	2.96	1.97
NGC 2403	3.25	7.90	2.05	3.10	3.31	1.83	2.78
NGC 3198	9.36	9.00	2.72	5.00	2.67	3.95	2.07
NGC 2903	6.40	15.30	2.02	2.40	4.86	3.53	3.80
NGC 7331	14.90	54.00	4.48	11.30	5.51	4.72	3.73
NGC 2841	18.00	74.20	4.53	15.90	3.81	2.94	3.20